

## Indices

i)  $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

ii)  $9^0 = 1$

2i)  $(5a^2b)^3 \times 2b^4 = 125a^6b^3 \times 2b^4 = 250a^6b^7$

ii)  $\left(\frac{1}{16}\right)^{-1} = \left(\frac{16}{1}\right)^1 = 16$  (negative index is the reciprocal)

3.  $\frac{(3xy^4)^3}{6x^5y^2} = \frac{27x^3y^{12}}{6x^5y^2} = \frac{9y^{10}}{2x^2}$  or  $\frac{9}{2}x^{-2}y^{10}$

4i)  $\left(\frac{1}{4}\right)^0 = 1$

ii)  $16^{-\frac{3}{2}} = \frac{1}{16^{\frac{3}{2}}} = \frac{1}{\sqrt{16}^3} = \frac{1}{4^3} = \frac{1}{64}$

5.  $\left(\frac{1}{2}\right)^{-5} = \left(\frac{2}{1}\right)^5 = 32$

6.  $\left(\frac{1}{25}\right)^{-\frac{1}{2}} = \left(\frac{25}{1}\right)^{\frac{1}{2}} = \sqrt{25} = 5$

7i)  $25^{\frac{3}{2}} = \sqrt{25}^3 = 5^3 = 125$

ii)  $\left(\frac{7}{3}\right)^{-2} = \left(\frac{3}{7}\right)^2 = \frac{9}{49}$

8i)  $\left(\frac{9}{16}\right)^{-\frac{1}{2}} = \left(\frac{16}{9}\right)^{\frac{1}{2}} = \sqrt{\frac{16}{9}} = \frac{4}{3}$

ii)  $\frac{(2ac^2)^3 \times 9a^2c}{36a^4c^{12}} = \frac{8a^3c^6 \times 9a^2c}{36a^4c^{12}} = \frac{2a}{c^5}$  or  $2ac^{-5}$

9i)  $125\sqrt{5} = 5^3 \times 5^{\frac{1}{2}} = 5^{\frac{7}{2}}$

ii)  $(4a^3b^5)^2 = 16a^6b^{10}$

## Surds

$$1i \frac{\sqrt{48}}{2\sqrt{27}} = \frac{\sqrt{16}\sqrt{3}}{2\sqrt{9}\sqrt{3}} = \frac{4\sqrt{3}}{2 \times 3\sqrt{3}} = \frac{2}{3}$$

$$1i \quad (5-3\sqrt{2})^2 = (5-3\sqrt{2})(5-3\sqrt{2}) = 25 - 15\sqrt{2} - 15\sqrt{2} + 9 \times 2 = 43 - 30\sqrt{2}$$

$3\sqrt{2} \times 3\sqrt{2} = 3 \times 3 \times \sqrt{2} \times \sqrt{2} = 9 \times 2$   $\nearrow$

$$2i \quad \sqrt{75} + \sqrt{48} = \sqrt{25}\sqrt{3} + \sqrt{16}\sqrt{3} = 5\sqrt{3} + 4\sqrt{3} = 9\sqrt{3}$$

$$ii \quad \frac{14}{3-\sqrt{2}} = \frac{14(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})} = \frac{42+14\sqrt{2}}{9-2} = \frac{42+14\sqrt{2}}{7} = 6+2\sqrt{2}$$

$$3i \quad \sqrt{98} - \sqrt{50} = \sqrt{49}\sqrt{2} - \sqrt{25}\sqrt{2} = 7\sqrt{2} - 5\sqrt{2} = 2\sqrt{2}$$

$$ii \quad \frac{6\sqrt{5}}{2+\sqrt{5}} = \frac{6\sqrt{5}(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})} = \frac{12\sqrt{5} - 6 \times 5}{4-5} = \frac{-30+12\sqrt{5}}{-1} = 30-12\sqrt{5}$$

$$4i \quad \sqrt{48} + \sqrt{27} = \sqrt{16}\sqrt{3} + \sqrt{9}\sqrt{3} = 4\sqrt{3} + 3\sqrt{3} = 7\sqrt{3}$$

$$ii \quad \frac{5\sqrt{2}}{3-\sqrt{2}} = \frac{5\sqrt{2}(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})} = \frac{15\sqrt{2} + 5 \times 2}{9-2} = \frac{10+15\sqrt{2}}{7}$$

$$5i \quad \frac{1}{5+\sqrt{3}} = \frac{1(5-\sqrt{3})}{(5+\sqrt{3})(5-\sqrt{3})} = \frac{5-\sqrt{3}}{25-3} = \frac{5-\sqrt{3}}{22}$$

$$ii \quad (3-2\sqrt{7})^2 = (3-2\sqrt{7})(3-2\sqrt{7}) = 9 - 6\sqrt{7} - 6\sqrt{7} + 4 \times 7 = 37 - 12\sqrt{7}$$

$$6 \quad a+b+c = \frac{3}{2} + \frac{9-\sqrt{17}}{4} + \frac{9}{4} = \frac{6+9-\sqrt{17}+9+\sqrt{17}}{4} = \frac{24}{4} = 6$$

$$abc = \frac{3}{2} \left( \frac{9-\sqrt{17}}{4} \right) \left( \frac{9+\sqrt{17}}{4} \right) = \frac{3}{2} \left( \frac{81-17}{16} \right) = \frac{3}{2} \times \frac{64}{16} = 6$$

$$\therefore a+b+c = abc = 6 \quad \text{QED}$$

## Algebraic fractions

$$1 \quad x^2 - 4 = (x+2)(x-2)$$

$$x^2 - 5x + 6 = (x-2)(x-3)$$

$$\frac{x^2 - 4}{x^2 - 5x + 6} = \frac{(x+2)(x-2)}{(x-2)(x-3)} = \frac{x+2}{x-3}$$

$$2 \quad 3x^2 - 7x + 4 = 3x^2 - 3x - 4x + 4 \\ = 3x(x-1) - 4(x-1) \\ = (3x-4)(x-1)$$

$$x^2 - 1 = (x+1)(x-1)$$

$$\frac{3x^2 - 7x + 4}{x^2 - 1} = \frac{(3x-4)(x-1)}{(x+1)(x-1)} = \frac{3x-4}{x+1}$$

## Proof

$$1. n^2 + n = n(n+1)$$

or

$$n^2 + n$$

when  $n$  is odd,  $n+1$  is even  
odd  $\times$  even = even

when  $n$  is even,  $n+1$  is odd  
even  $\times$  odd = even     $\square$

when  $n$  is odd,  $n^2$  is odd  
odd + odd = even

when  $n$  is even,  $n^2$  is even  
even + even = even     $\square$

$$2. n^3 - n = n(n^2 - 1)$$

or

$$n^3 - n$$

when  $n$  is odd,  $n^2$  is odd  
 $n^2 - 1$  is even  
odd  $\times$  even = even

when  $n$  is even,  $n^2$  is even  
 $n^2 - 1$  is odd  
even  $\times$  odd = even     $\square$

when  $n$  is odd,  $n^3$  is odd  
odd - odd = even

when  $n$  is even,  $n^3$  is even  
even - even = even     $\square$

3.  $n$  is even, let  $n = 2m$

$$\begin{aligned} 3n^2 + 6n &= 3(2m)^2 + 6(2m) \\ &= 12m^2 + 12m \\ &= 12(m^2 + m) \end{aligned}$$

$\therefore 12$  is a factor when  $n$  is even     $\square$

ii try  $n$  is odd, let  $n = 2m + 1$

$$\begin{aligned} 3n^2 + 6n &= 3(2m+1)^2 + 6(2m+1) \\ &= 3(4m^2 + 4m + 1) + 6(2m+1) \\ &= 12m^2 + 12m + 3 + 12m + 6 \\ &= 12m^2 + 24m + 9 \\ &= 12(m^2 + 2m) + 9 \end{aligned}$$

$\therefore 12$  is not a factor when  $n$  is odd

$$\begin{aligned} 4. n^3 + 3n^2 + 2n &= n(n^2 + 3n + 2) \\ &= n(n+1)(n+2) \end{aligned}$$

this is the product of 3 consecutive numbers

- at least one must be even
- one must be a multiple of 3

a multiple of 2  $\times$  a multiple of 3 = a multiple of 6     $\square$

## Solving linear inequalities

$$\begin{aligned} 1. \quad & 6(x+3) > 2x+5 \\ & 6x+18 > 2x+5 \\ & 4x > -13 \\ & x > -\frac{13}{4} \end{aligned}$$

$$\begin{aligned} 2. \quad & 3x-1 > 5-x \\ & 4x > 6 \\ & x > \frac{3}{2} \end{aligned}$$

$$\begin{aligned} 3. \quad & \frac{5x-3}{2} < x+5 \\ & 5x-3 < 2x+10 \\ & 3x < 13 \\ & x < \frac{13}{3} \end{aligned}$$

$$\begin{aligned} 4. \quad & \frac{3(2x+1)}{4} > -6 \\ & 3(2x+1) > -24 \\ & 6x+3 > -24 \\ & 6x > -27 \\ & x > -\frac{9}{2} \end{aligned}$$

$$\begin{aligned} 5. \quad & 7-x < 5x-2 \\ & -6x < -9 \\ & 6x > 9 \\ & x > \frac{3}{2} \end{aligned}$$

$$\begin{aligned} 6. \quad & 1-2x < 4+3x \\ & -5x < 3 \\ & 5x > -3 \\ & x > -\frac{3}{5} \end{aligned}$$

## Solving equations

$$1 \quad \frac{4x+5}{2x} = -3$$

$$4x+5 = -6x$$

$$10x = -5$$

$$x = -\frac{1}{2}$$

$$2 \quad \frac{3x+1}{2x} = 4$$

$$3x+1 = 8x$$

$$5x = 1$$

$$x = \frac{1}{5}$$

$$3 \quad y^2 - 7y + 12 = 0$$

$$(y-3)(y-4) = 0$$

$$\therefore y = 3 \text{ or } y = 4$$

$$x^4 - 7x^2 + 12 = 0$$

$$(x^2)^2 - 7(x^2) + 12 = 0$$

$$\text{let } y = x^2$$

$$y^2 - 7y + 12 = 0$$

$$\therefore y = 3 \text{ or } y = 4$$

$$\therefore x^2 = 3 \text{ or } x^2 = 4$$

$$x = \pm\sqrt{3} \text{ or } x = \pm 2$$

$$4 \quad 4x^2 + 20x + 25 = 0$$

$$4x^2 + 10x + 10x + 25 = 0$$

$$2x(2x+5) + 5(2x+5) = 0$$

$$(2x+5)(2x+5) = 0$$

$$\therefore x = -\frac{5}{2}$$

$$5 \quad 2x^2 + 3x = 0$$

$$x(2x+3) = 0$$

$$\therefore x = 0 \text{ or } x = -\frac{3}{2}$$

## Forming and solving equations

i. Area of triangle =  $\frac{bh}{2}$

$$= \frac{(2x-3)(x+1)}{2} = 9$$

$$(2x-3)(x+1) = 18$$

$$2x^2 - x - 3 = 18$$

$$2x^2 - x - 21 = 0 \quad \text{QED}$$

ii.  $2x^2 - x - 21 = 0$

$$2x^2 + 6x - 7x - 21 = 0$$

$$2x(x+3) - 7(x+3) = 0$$

$$(2x-7)(x+3) = 0$$

$$\therefore x = \frac{7}{2} \text{ or } x = -3$$

↑ this would give negative lengths  $\therefore$  not possible

$$\text{height} = x+1 = \frac{7}{2} + 1 = 4.5 \text{ cm}$$

$$\text{base} = 2x-3 = 2 \times \frac{7}{2} - 3 = 4 \text{ cm}$$

2. Area of trapezium =  $\frac{(a+b)h}{2}$

$$= \frac{(x+2)+(3x+6)}{2} \times 2x = 140$$

$$\left( \frac{4x+8}{2} \right) 2x = 140$$

$$4x^2 + 8x = 140$$

$$4x^2 + 8x - 140 = 0$$

$$x^2 + 2x - 35 = 0 \quad \text{QED}$$

ii.  $x^2 + 2x - 35 = 0$

$$(x-5)(x+7) = 0$$

$$\therefore x = 5 \text{ or } x = -7$$

↑ this would give negative lengths  $\therefore$  not possible

$$AB = 3x+6 = 3 \times 5 + 6 = 21 \text{ cm}$$

## Completing the square and turning points

$$\begin{aligned} \text{1i. } & \underbrace{x^2 + 6x + 5} \\ & \downarrow \\ & = (x+3)^2 - 9 + 5 \\ & = (x+3)^2 - 4 \end{aligned}$$

$$\begin{aligned} (x+3)^2 &= x^2 + 6x + 9 \\ (x+3)^2 - 9 &= x^2 + 6x \end{aligned}$$

ii coordinates of minimum point =  $(-3, -4)$

$$\begin{aligned} \text{2i. } & \underbrace{x^2 - 7x + 6} \\ & \downarrow \\ & = (x - \frac{7}{2})^2 - \frac{49}{4} + 6 \\ & = (x - \frac{7}{2})^2 - \frac{25}{4} \end{aligned}$$

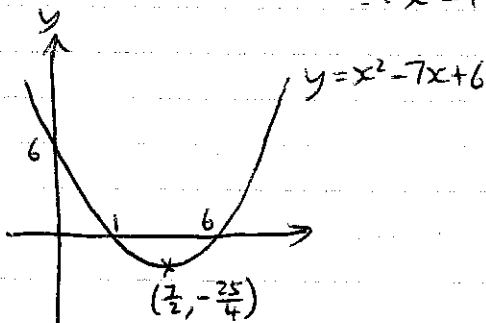
$$\begin{aligned} (x - \frac{7}{2})^2 &= x^2 - 7x + \frac{49}{4} \\ (x - \frac{7}{2})^2 - \frac{49}{4} &= x^2 - 7x \end{aligned}$$

ii coordinates of minimum point =  $(\frac{7}{2}, -\frac{25}{4})$

iii crosses y-axis at  $y=6$

$$\begin{aligned} \text{crosses } x\text{-axis when } & x^2 - 7x + 6 = 0 \\ & (x-1)(x-6) = 0 \end{aligned}$$

$$\therefore x = 1 \text{ or } x = 6$$



$$\begin{aligned} \text{3i. } & 3x^2 + 6x + 10 \\ & = 3(x^2 + 2x) + 10 \\ & \downarrow \\ & = 3[(x+1)^2 - 1] + 10 \\ & = 3(x+1)^2 - 3 + 10 \\ & = 3(x+1)^2 + 7 \end{aligned}$$

$$\begin{aligned} (x+1)^2 &= x^2 + 2x + 1 \\ (x+1)^2 - 1 &= x^2 + 2x \end{aligned}$$

ii minimum value of  $y = 7$

$\therefore$  always above x-axis

QED



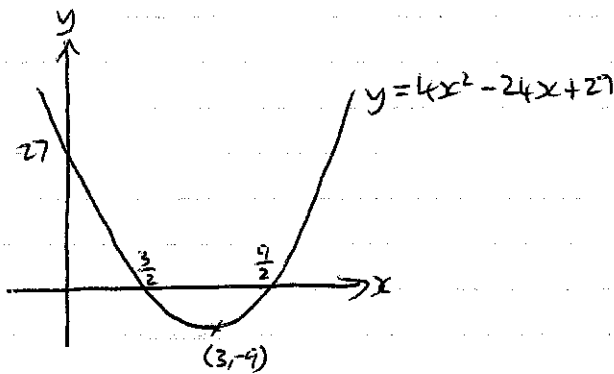
$$\begin{aligned}
 \text{4i. } & 4x^2 - 24x + 27 \\
 & = 4(x^2 - 6x) + 27 \\
 & \quad \downarrow \\
 & = 4[(x-3)^2 - 9] + 27 \\
 & = 4(x-3)^2 - 36 + 27 \\
 & = 4(x-3)^2 - 9
 \end{aligned}$$

$$\begin{aligned}
 (x-3)^2 & = x^2 - 6x + 9 \\
 (x-3)^2 - 9 & = x^2 - 6x
 \end{aligned}$$

ii. coordinates of minimum point =  $(3, -9)$

$$\begin{aligned}
 \text{iii. } & 4x^2 - 24x + 27 = 0 \\
 & 4(x-3)^2 - 9 = 0 \\
 & 4(x-3)^2 = 9 \\
 & (x-3)^2 = \frac{9}{4} \\
 & x-3 = \pm \frac{3}{2} \\
 & x = 3 \pm \frac{3}{2}
 \end{aligned}$$

$$\therefore x = \frac{3}{2} \text{ or } x = \frac{9}{2}$$



## Discriminant and roots

$$1. \quad b^2 - 4ac = (5)^2 - 4(3)(2) \\ = 1$$

$\therefore$  2 distinct real roots

2. no real roots

$$\therefore \quad b^2 - 4ac < 0 \\ (3)^2 - 4(2)(-k) < 0 \\ 9 + 8k < 0 \\ 8k < -9 \\ k < -\frac{9}{8}$$

3. no real roots

$$\therefore \quad b^2 - 4ac < 0 \\ k^2 - 4(2)(2) < 0 \\ k^2 - 16 < 0 \\ k^2 < 16 \\ -4 < k < 4$$

4. for intersection, solve simultaneously

$$x^2 - 5x + 7 = 3x - 10 \\ x^2 - 8x + 17 = 0$$

$$b^2 - 4ac = (-8)^2 - 4(1)(17) \\ = -4 < 0$$

$\therefore$  no solution

$\therefore$  lines do not intersect  $\quad \square \Rightarrow$

## Changing the subject of a formula

$$\begin{aligned} 1 \quad s &= ut + \frac{1}{2}at^2 \\ \frac{1}{2}at^2 &= s - ut \\ a &= \frac{2(s-ut)}{t^2} \end{aligned}$$

$$\begin{aligned} 2 \quad V &= \frac{1}{3}\pi r^2 h \\ r^2 &= \frac{3V}{\pi h} \\ r &= \pm \sqrt{\frac{3V}{\pi h}} \end{aligned}$$

$$\begin{aligned} 3 \quad a &= \frac{\sqrt{y}-5}{c} \\ \sqrt{y}-5 &= ac \\ \sqrt{y} &= ac+5 \\ y &= (ac+5)^2 \end{aligned}$$

$$\begin{aligned} 4 \quad c &= \sqrt{\frac{a+b}{2}} \\ \frac{a+b}{2} &= c^2 \\ a+b &= 2c^2 \\ a &= 2c^2 - b \end{aligned}$$

$$\begin{aligned} 5 \quad V &= \frac{1}{3}\pi r^2 \sqrt{l^2 - r^2} \\ \sqrt{l^2 - r^2} &= \frac{3V}{\pi r^2} \\ l^2 - r^2 &= \left(\frac{3V}{\pi r^2}\right)^2 \\ l^2 &= \left(\frac{3V}{\pi r^2}\right)^2 + r^2 \\ l &= \pm \sqrt{\left(\frac{3V}{\pi r^2}\right)^2 + r^2} \end{aligned}$$

continued →

$$\begin{aligned}
 6 \quad 2a+5c &= af+7c \\
 2a-af &= 7c-5c \\
 a(2-f) &= 2c \\
 a &= \frac{2c}{2-f}
 \end{aligned}$$

$$\begin{aligned}
 7 \quad y+5 &= x(y+2) \\
 y+5 &= xy+2x \\
 y-xy &= 2x-5 \\
 y(1-x) &= 2x-5 \\
 y &= \frac{2x-5}{1-x}
 \end{aligned}$$

$$\begin{aligned}
 \text{or} \quad xy-y &= 5-2x \\
 y(x-1) &= 5-2x \\
 y &= \frac{5-2x}{x-1}
 \end{aligned}$$

$$\begin{aligned}
 8 \quad P &= \frac{C}{C+4} \\
 P(C+4) &= C \\
 PC+4P &= C \\
 C-PC &= 4P \\
 C(1-P) &= 4P \\
 C &= \frac{4P}{1-P}
 \end{aligned}$$

$$\begin{aligned}
 \text{or} \quad PC-C &= -4P \\
 C(P-1) &= -4P \\
 C &= -\frac{4P}{P-1}
 \end{aligned}$$

$$\begin{aligned}
 9 \quad y &= \frac{1-2x}{x+3} \\
 y(x+3) &= 1-2x \\
 xy+3y &= 1-2x \\
 xy+2x &= 1-3y \\
 x(y-2) &= 1-3y \\
 x &= \frac{1-3y}{y-2}
 \end{aligned}$$

## Equation of a straight line

1  $y = 5x - 4$ , parallel line  $y = 5x + c$  when  $x = 2, y = 13$   
 $13 = 5 \times 2 + c$   
 $c = 3$   
 $\therefore y = 5x + 3$

2  $y = 3x + 1$ , parallel line  $y = 3x + c$  when  $x = 4, y = 5$   
 $5 = 3 \times 4 + c$   
 $c = -7$   
 $\therefore y = 3x - 7$

3 gradient,  $m = \frac{11 - -9}{3 - -1} = \frac{20}{4} = 5$

$y = 5x + c$  when  $x = 3, y = 11$   
 $11 = 5 \times 3 + c$   
 $c = -4$   
 $\therefore y = 5x - 4$

4  $3x + 2y = 6$

$2y = -3x + 6$

$y = -\frac{3}{2}x + 3$ , parallel line  $y = -\frac{3}{2}x + c$  when  $x = 2, y = 10$   
 $10 = -\frac{3}{2} \times 2 + c$   
 $c = 13$

$\therefore y = -\frac{3}{2}x + 13$  or  $3x + 2y = 26$

5i gradient,  $m = \frac{9 - 1}{3 - -1} = \frac{8}{4} = 2$

$y = 2x + c$  when  $x = 3, y = 9$   
 $9 = 2 \times 3 + c$   
 $c = 3$   
 $\therefore y = 2x + 3$

ii gradient,  $-\frac{1}{m} = -\frac{1}{2}$

coordinates of midpoint =  $\left(\frac{-1+3}{2}, \frac{1+9}{2}\right) = (1, 5)$

$y = -\frac{1}{2}x + c$  when  $x = 1, y = 5$   
 $5 = -\frac{1}{2} \times 1 + c$   
 $c = \frac{11}{2}$   
 $\therefore y = -\frac{1}{2}x + \frac{11}{2}$   
 $2y + x = 11$  QED

## Intersection of two lines

$$1 \quad \begin{array}{l} y = 3x + 1 \quad \text{--- ①} \\ x + 3y = 6 \quad \text{--- ②} \end{array}$$

sub. ① into ②

$$x + 3(3x + 1) = 6$$

$$x + 9x + 3 = 6$$

$$10x = 3$$

$$x = \frac{3}{10}$$

sub.  $x = \frac{3}{10}$  into ①

$$y = 3 \times \frac{3}{10} + 1$$

$$= \frac{19}{10}$$

coordinates of point of intersection =  $(\frac{3}{10}, \frac{19}{10})$

$$2 \quad \begin{array}{l} y = 2x - 5 \quad \text{--- ①} \\ 6x + 2y = 7 \quad \text{--- ②} \end{array}$$

sub. ① into ②

$$6x + 2(2x - 5) = 7$$

$$6x + 4x - 10 = 7$$

$$10x = 17$$

$$x = \frac{17}{10}$$

sub.  $x = \frac{17}{10}$  into ①

$$y = 2 \times \frac{17}{10} - 5$$

$$= -\frac{8}{5}$$

coordinates of point of intersection =  $(\frac{17}{10}, -\frac{8}{5})$

$$3 \quad y = x^2 - 6x + 2 \quad \text{--- ①}$$

$$y = 2x - 14 \quad \text{--- ②}$$

equate ① and ②

$$x^2 - 6x + 2 = 2x - 14$$

$$x^2 - 8x + 16 = 0$$

$$(x-4)^2 = 0$$

$$\therefore x = 4$$

sub.  $x=4$  into ②

$$y = 2 \times 4 - 14$$

$$= -6$$

only one point of intersection  $(4, -6)$

$\therefore y = 2x - 14$  is a tangent to  $y = x^2 - 6x + 2$  QED

$$4 \quad y = 4x^2 + 24x + 31 \quad \text{--- ①}$$

$$x + y = 10 \quad \text{--- ②}$$

from ②,  $y = 10 - x$  --- ③

equate ① and ③

$$\therefore 4x^2 + 24x + 31 = 10 - x$$

$$4x^2 + 25x + 21 = 0$$

$$4x^2 + 4x + 21x + 21 = 0$$

$$4x(x+1) + 21(x+1) = 0$$

$$(4x+21)(x+1) = 0$$

$$\therefore x = -\frac{21}{4} \quad \text{or} \quad x = -1$$

sub.  $x = -\frac{21}{4}$  into ③

$$y = 10 - \left(-\frac{21}{4}\right) = \frac{61}{4}$$

$$= \frac{61}{4}$$

sub.  $x = -1$  into ③

$$y = 10 - (-1)$$

$$= 11$$

coordinates of point of intersection =  $\left(-\frac{21}{4}, \frac{61}{4}\right)$  and  $(-1, 11)$

$$5. \quad x^2 + y^2 = 25 \quad \text{--- ①}$$

$$y = 3x \quad \text{--- ②}$$

sub. ② into ①

$$x^2 + (3x)^2 = 25$$

$$x^2 + 9x^2 = 25$$

$$10x^2 = 25$$

$$x^2 = \frac{5}{2}$$

$$x = \pm \sqrt{\frac{5}{2}} = \pm \frac{\sqrt{10}}{2}$$

$$\sqrt{\frac{5}{2}} = \frac{\sqrt{5}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{10}}{2}$$

sub.  $x = -\frac{\sqrt{10}}{2}$  into ②

$$y = -\frac{3\sqrt{10}}{2}$$

sub.  $x = \frac{\sqrt{10}}{2}$  into ②

$$y = \frac{3\sqrt{10}}{2}$$

coordinates of point of intersection =  $(-\frac{\sqrt{10}}{2}, -\frac{3\sqrt{10}}{2})$  and  $(\frac{\sqrt{10}}{2}, \frac{3\sqrt{10}}{2})$

$$6i. \quad x^2 + y^2 = 45 \quad \text{--- ①}$$

centre =  $(0, 0)$  and radius =  $\sqrt{45} = 3\sqrt{5}$

$$ii. \quad x + y = 3 \quad \text{--- ②}$$

from ②,  $y = 3 - x \quad \text{--- ③}$

sub. ③ into ①

$$x^2 + (3-x)^2 = 45$$

$$x^2 + x^2 - 6x + 9 = 45$$

$$2x^2 - 6x - 36 = 0$$

$$x^2 - 3x - 18 = 0$$

$$(x+3)(x-6) = 0$$

$$\therefore x = -3 \text{ or } x = 6$$

sub  $x = -3$  into ③

$$y = 3 - (-3)$$

$$= 6$$

sub.  $x = 6$  into ③

$$y = 3 - 6$$

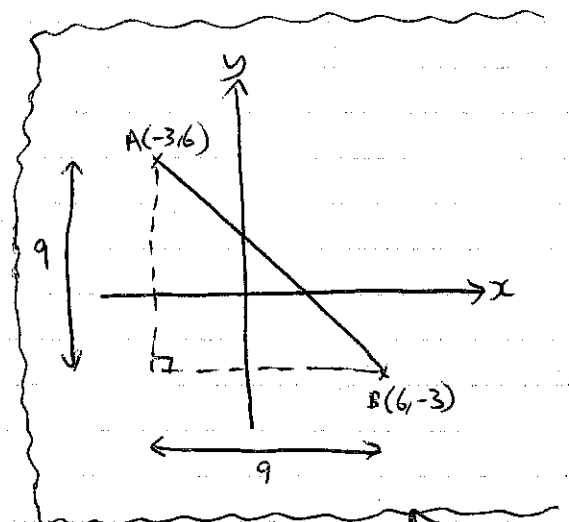
$$= -3$$

coordinates of point of intersection =  $(-3, 6)$  and  $(6, -3)$

$$AB^2 = 9^2 + 9^2$$

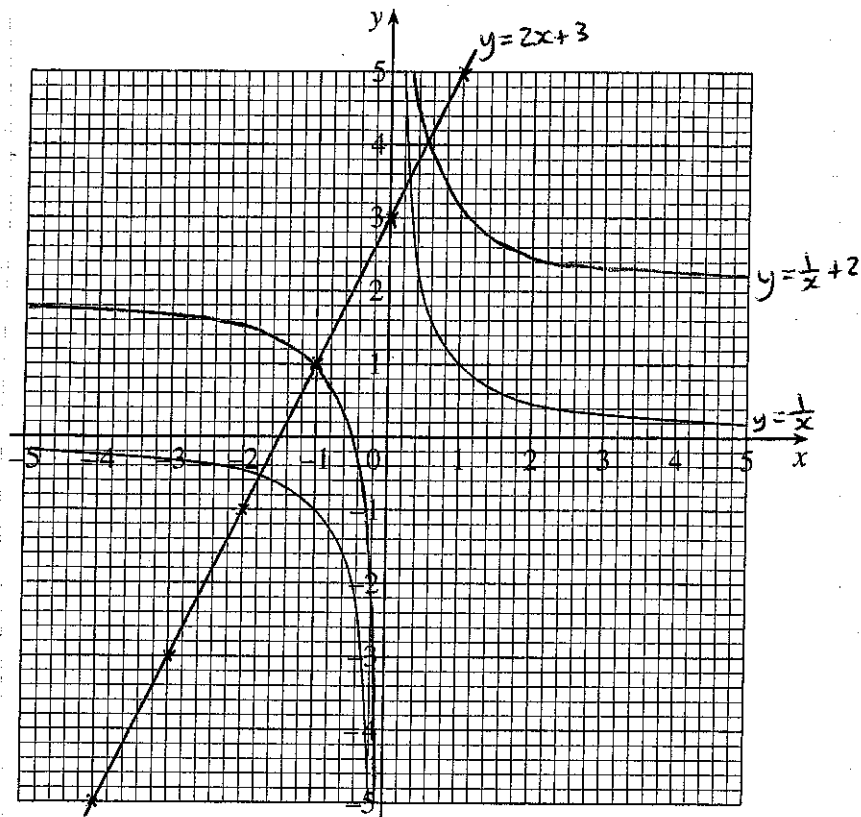
$$AB = \sqrt{162}$$

QED





## Using graphs to solve equations



i  $\frac{1}{x} = 2x + 3$

intersection of  $y = \frac{1}{x}$  and  $y = 2x + 3$

$$x \approx -1.8 \quad \text{or} \quad x \approx 0.3$$

ii  $\frac{1}{x} = 2x + 3$

$$1 = 2x^2 + 3x$$

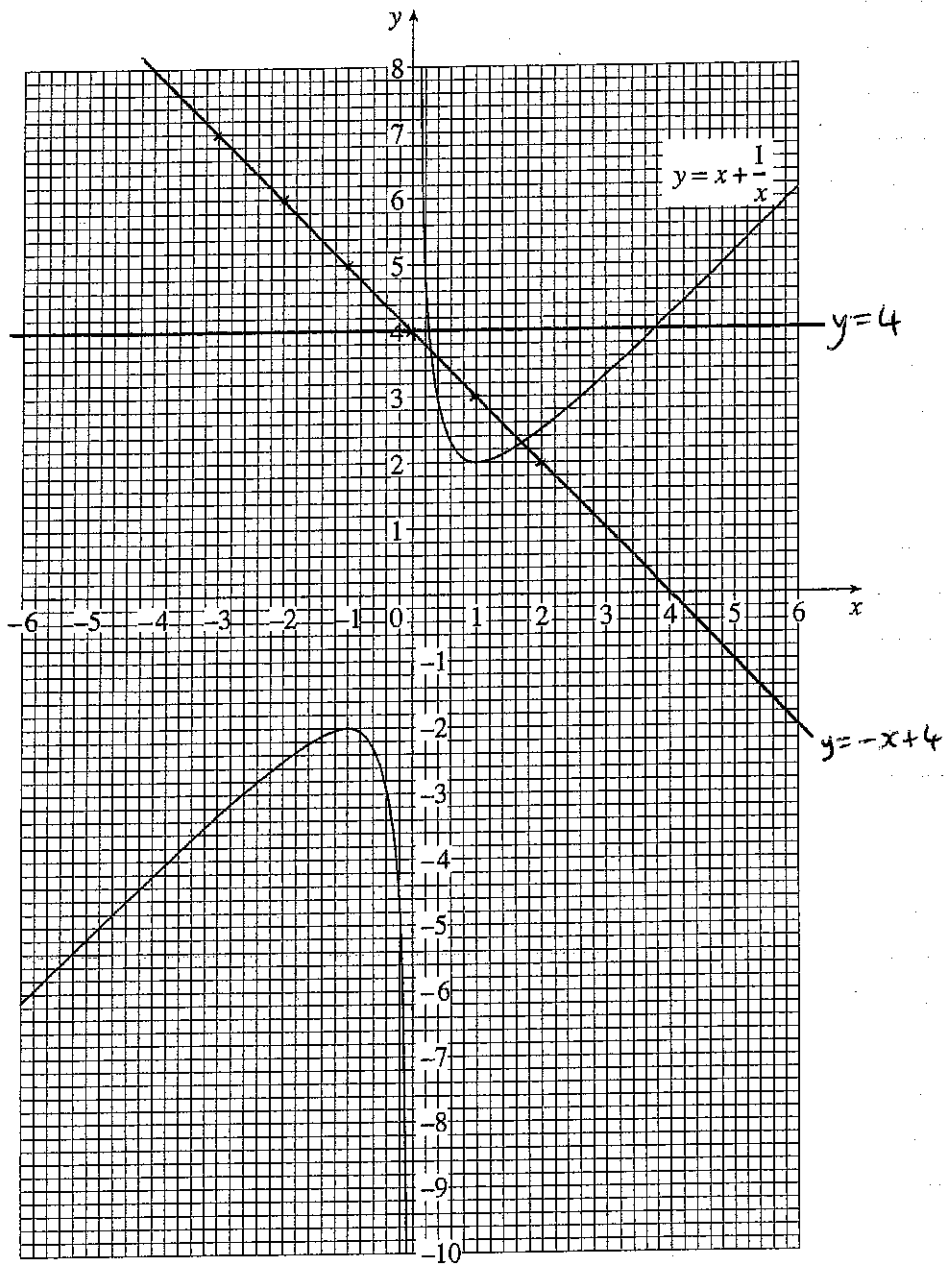
$$2x^2 + 3x - 1 = 0$$

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(2)(-1)}}{2(2)}$$
$$= \frac{-3 \pm \sqrt{17}}{4}$$

iii  $y = \frac{1}{x} + 2$

translate graph of  $y = \frac{1}{x}$  by vector  $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$

2



A.  $x + \frac{1}{x} = 4$

intersection of  $y = x + \frac{1}{x}$  and  $y = 4$

$x \approx 0.3$  or  $x \approx 3.7$

B.  $2x + \frac{1}{x} = 4$

$x + \frac{1}{x} = -x + 4$

intersection of  $y = x + \frac{1}{x}$  and  $y = -x + 4$

$x \approx 0.3$  or  $x \approx 1.7$

## Transformation of graphs

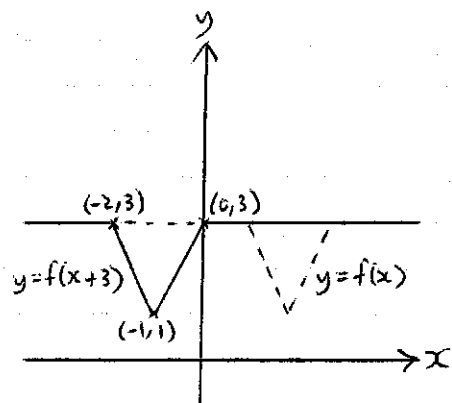
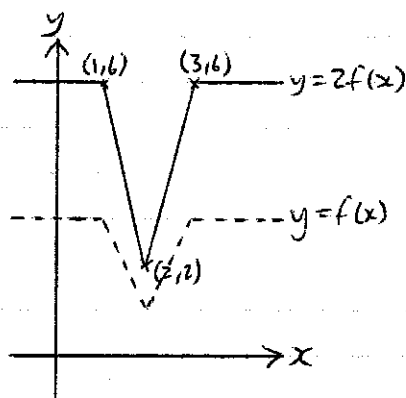
i.  $(10, 4)$

ii.  $(5, 11)$

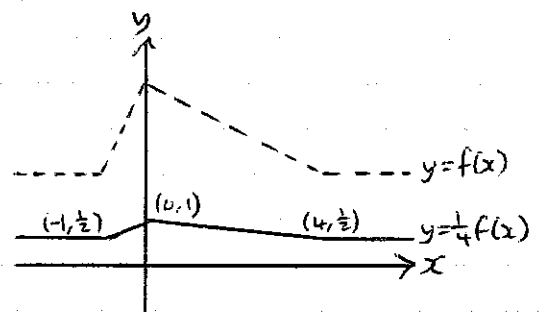
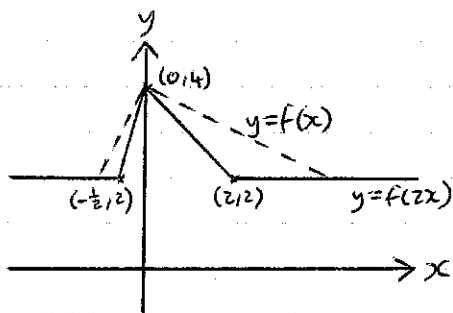
2. i.  $(3, 15)$

ii.  $(\frac{3}{2}, 5)$

3. i



4. i



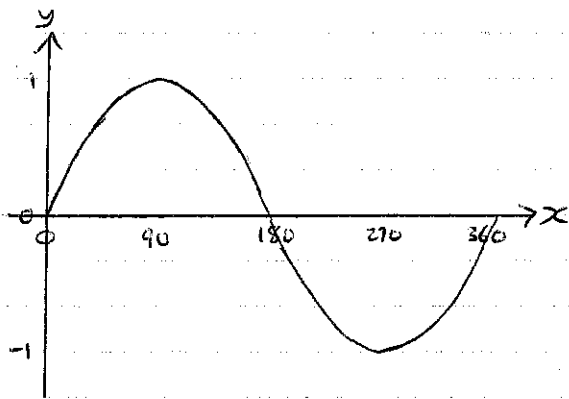
5. i.  $y = 2f(x)$

ii.  $y = f(x-3)$

6.  $y = (x-2)^2 - 4$

7. translation by vector  $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$

# Trigonometric graphs

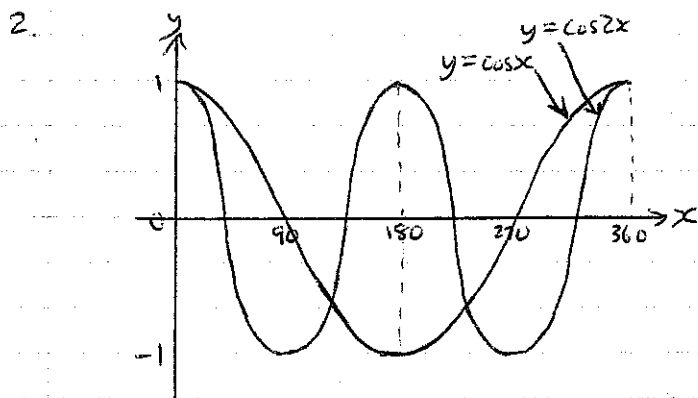


$$\begin{aligned}\sin x &= -0.68 \\ x &= \sin^{-1}(-0.68) \\ &= -42.8^\circ\end{aligned}$$

$$\begin{aligned}\text{or } x &= 180 - (-42.8) && \text{by symmetry} \\ &= 222.8^\circ\end{aligned}$$

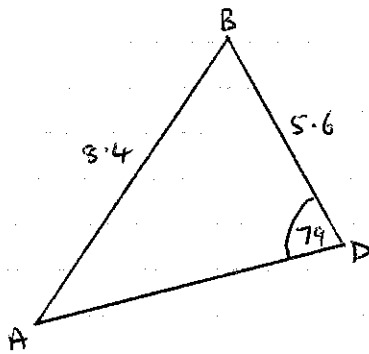
$$\begin{aligned}\text{or } x &= -42.8 + 360 && \text{periodic } (360^\circ) \\ &= 317.2^\circ\end{aligned}$$

$$\therefore x = 222.8^\circ \text{ or } 317.2^\circ$$



## Sine rule and cosine rule

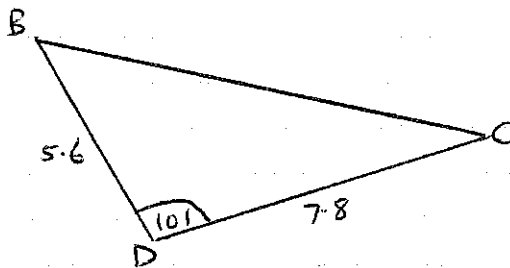
i.



$$\frac{\sin BAD}{5.6} = \frac{\sin 79}{8.4}$$

$$BAD = \sin^{-1} \left( \frac{5.6 \sin 79}{8.4} \right)$$
$$= 40.9^\circ$$

ii.

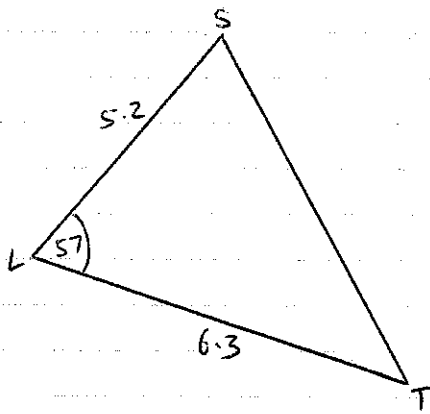


$$BC^2 = (5.6)^2 + (7.8)^2 - 2(5.6)(7.8) \cos 101^\circ$$

$$BC = 10.4 \text{ cm}$$

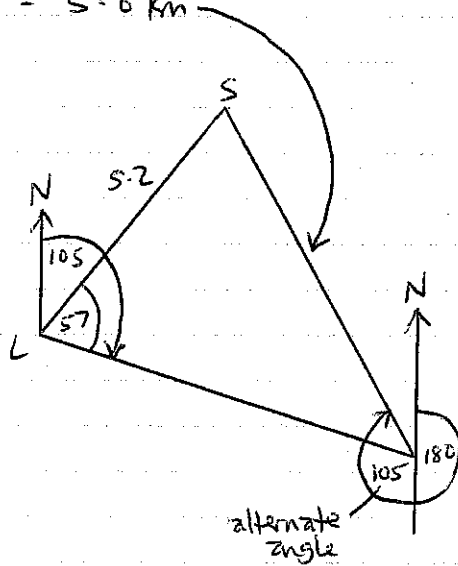
continued →

2i



$$ST^2 = (5.2)^2 + (6.3)^2 - 2(5.2)(6.3) \cos 57^\circ$$
$$ST = 5.6 \text{ km}$$

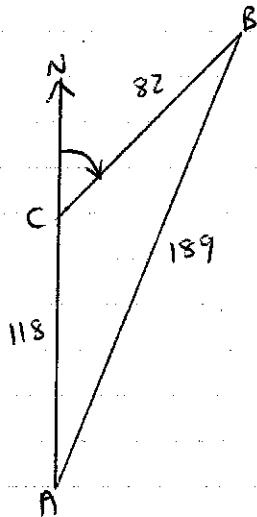
ii



$$\frac{\sin LTS}{5.2} = \frac{\sin 57}{5.5718\dots}$$
$$LTS = \sin^{-1} \left( \frac{5.2 \sin 57}{5.5718\dots} \right)$$
$$= 51.5^\circ$$

$$\text{bearing of S from T} = 180 + 105 + 51.5$$
$$= 336.5^\circ \quad \text{or} \quad 337^\circ$$

3i



$$(189)^2 = (82)^2 + (118)^2 - 2(82)(118) \cos ACB$$

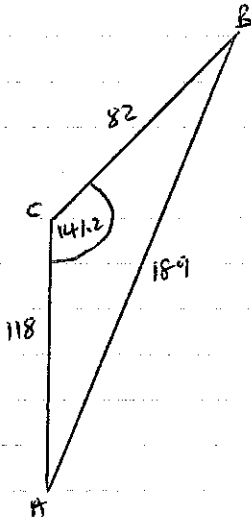
$$\cos ACB = \frac{(82)^2 + (118)^2 - (189)^2}{2(82)(118)}$$

$$ACB = 141.2^\circ$$

$$\text{bearing of B from C} = 180 - 141.2$$

$$= 038.8^\circ \quad \text{or } 039^\circ$$

ii



$$\text{area} = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} (82)(118) \sin 141.2$$

$$= 3034.2 \text{ m}^2$$

## Arc length and sector area

$$1 \text{ arc length} = 2\pi r \times \frac{\theta}{360}$$

$$2\pi(18.0) \times \frac{\theta}{360} = 43.2$$

$$\theta = \frac{43.2 \times 360}{2\pi(18.0)}$$
$$= 137.5^\circ$$

$$2 \text{ sector area} = \pi r^2 \times \frac{\theta}{360}$$

$$\pi(5)^2 \times \frac{\theta}{360} = 9$$

$$\theta = \frac{9 \times 360}{\pi(5)^2}$$
$$= 41.3^\circ$$